**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

* Time for car drop x = 60-10 = 50, *μ* = 45, *σ* = 8

Probability that the service manager cannot meet his commitment is,

P(x) = 1 - stats.norm.cdf(50,45,8) = **0.2659**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

* **False**.

P(x > 44) = 1 - stats.norm.cdf(44,38,6) = 0.1586

P(38 < X < 44) = stats.norm.cdf(44,38,6) - stats.norm.cdf(38,38,6) = 0.3413

As probability of ‘employees between 38 and 44’ is more than that of ‘employees older than 44’, statement above is False.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

* **True**.

Probability of employees under 30 is,

P(x < 30) = stats.norm.cdf(30,38,6) = 0.0912

No. of employees attracted = 400 \* stats.norm.cdf(30,38,6) = 36.4844

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

* **2X1 = N(2μ, 4σ2)** and **X1 + X2 = N(2μ, σ2)**

From above calculations we can say that spread of data range in 1st case is 4 times that of 2nd case whereas mean in both cases are same.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

* Interval For probability of 0.99 is,

stats.norm.interval(0.99,100,20) = **(48.48, 151.51)**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

* Combined distributon of Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) can be given as,

Mean = 5 + 7 = 12, SD2 = 32 + 42 = 9+16 = 25 and SD = √25 = 5

In Rupee, mean = 12 \* 45 = **540** and SD = 5 \* 45 = **225**

Rupee range such that it contains 95% probability is,

stats.norm.interval(0.95,540,225) = **(99.00, 980.99)**.

1. Specify the 5th percentile of profit (in Rupees) for the company

* Percentile value = μ + zσ

Where, μ = 540, σ = 225, z = z-score corresponding to percentile value = -1.645

Percentile value = 540 + (-1.645) \* 225 = 169.875 ≈ **170 million Rs**.

1. Which of the two divisions has a larger probability of making a loss in a given year?

* Probability of making a loss for Division 1 is,

P(Loss1) = stats.norm.cdf(0,5,3) = **0.0478**.

Probability of making a loss for Division 2 is,

P(Loss2) = stats.norm.cdf(0,7,4) = **0.0400**.

From above calculations we can say that **Division 1** has a larger probability of making a loss in a given year than Division 2